STABLE DIRECT ADAPTIVE PERIODIC CONTROL OF NONMINIMUM-PHASE SYSTEMS

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Abstract

The main contribution of this paper is to put stability requirements for convergence of *direct* adaptive periodic controllers on equal footing with requirements for *indirect* adaptive periodic control, as set forth by Lozano [19]. The resulting stability conditions are simply that the plant order and delay are known a-priori. No prior knowledge of the plant high-frequency gain is used, and persistent excitation is not required. More importantly, no assumption or knowledge is required as to whether the plant is minimum or nonminimum phase.

1 INTRODUCTION

An intriguing property associated with generalized sampling mechanisms is their ability to relocate transmission zeros of the plant. The potential benefit of sampling for zero relocation was noted in the paper by Astrom, Hagander and Sternby [2]. Subsequent research investigated applications of generalized sampling mechanisms to such problems as robust control, simultaneous stabilization, sensitivity minimization, and zero placement, cf., [10][11][15][17].

Generalized sampling can take many different forms, e.g., multirate sampling, periodic control, generalized sample-and-hold, etc. Most approaches have an interpretation as a mathematical "lifting" where a serial to parallel conversion is performed on the plant input and output signals, and mappings are considered between the vectorized quantities.

In Lozano [19] an important lifting was introduced for which the transmission zeros are at the origin. Such liftings are denoted here as *zero annihilation* (ZA) liftings. General conditions characterizing the ZA property can be found in Bayard [5], along with several *extended horizon* lifting versions which satisfy the ZA conditions. Extended horizon liftings have the advantage of reducing required control torque and the size of the transient response, and have been applied to problems in optical instrument pointing [8], and structural vibration damping [7].

The transmission zeros of the ZA lifted plant are at the origin regardless of whether the original plant is minimum or nonminimum phase. This is important since it provides a means by which a nonminimum phase plant can be "transformed" into a minimum phase lifted plant. In light of this property, it is not surprising that several stable adaptive control approaches for nonminimum phase systems have been developed based on such liftings [4][19][21][21],

Of particular interest are the adaptive controllers of Lozano [19][21] [20], These adaptive controllers are of the *indirect* type, i.e., the plant parameters are estimated first, and are then used to compute the control gains. A main result of Lozano is that only the plant order and plant delay is required to be known to establish stability.

The present paper will consider *direct* adaptive control for the same class of liftings. The main contribution of this paper is to put stability requirements for convergence of direct adaptive periodic controllers on equal footing with requirements for indirect adaptive periodic control, i.e., that the plant order and delay are known a-priori. No prior knowledge of the plant high-frequency gain is used and persistent excitation is not required. More importantly, no assumption or knowledge is required as to whether the plant is minimum or nonminimum phase.

2 BACKGROUND

A brief overview of zero annihilating liftings is given in Appendix B. The plant representation after zero annihilating lifting is of the general form (cf., (B.4)),

$$Y_k = AY_{k-1} + HU_k \tag{2.1}$$

where $A \triangleq S_y A_a S_y^T \in R^{n_y \times n_y}$, $H \triangleq S_y H_a S_u^T \in R^{n_y \times n_u}$, the vector $U_k \in R^{n_u}$ is the lifted plant input and $Y_k \in R^{n_y}$ is the lifted plant output. For adaptive control purposes we will use Lozano's lifting [19] corresponding to (2.1) with $o_u = o_y = n$, and H nonsingular.

It is emphasized that any controllable and observable linear time-invariant plant can be lifted into the form (2.1) using only knowledge of its *plant order and delay* [19]. Furthermore, the *nonsingularity* of H is ensured simply by the controllability and observability of the original (unlifted) plant and does not depend on whether the true (unlifted) plant is minimum or nonminimum-phase.

The discussion will focus on developing a stable adaptive law for (2.1)

A rearrangement of (2.1) gives the equivalent plant representation,

Linear Control Form

$$U_k = KY_{k-1} + LY_k = \Theta r_k \tag{2.2}$$

where,

$$K = -H^{-1}A; \quad L = H^{-1} \tag{2.3}$$

$$\Theta = [K \mid L]; \quad r_k = [Y_{k-1}^T \mid Y_k^T]^T \tag{2.4}$$

Representation (2.2) is said to be in *Linear Control Form* (cf., Goodwin and Sin [14]) since the control is an unknown linear combination of measured signals. One important advantage is that a deadbeat controller can be written directly in terms of the gains K and L as follows,

Deadbeat Control

$$U_k^o = KY_{k-1} + LY_k^d = \Theta r_k^d \tag{2.5}$$

$$r_k^d = [Y_{k-1}^T \mid (Y_k^d)^T]^T \tag{2.6}$$

where Y_k^d is a specified trajectory to be tracked by Y_k. Hence, it is only necessary to estimate K and L in (2.2) and then "copy" the estimates for implementing the deadbeat control (2.5).

Lozano has developed several adaptive control approaches [18] [21][20], based on the representation (2.1). Lozano's approaches are "indirect" in the sense that the plant parameters A and H are first estimated from (2.1) and then used to compute the control gains K and L in (2.2) using the formulas in (2.3). From (2.3) it is seen that this requires a numerical inversion of the estimate of H each iteration. In order to ensure invertibility of this estimate, Lozano introduces a modification in [19] based on a polar decomposition.

In contrast to Lozano's approach, the present paper will focus on a "direct" adaptive scheme. In a direct scheme, the gains K and L in control law (2.5) are estimated directly from the plant representation (2.2). Earlier stable direct adaptation schemes have been developed for periodic control in Ortega [24] and Bayard [4]. The present direct adaptive approach is similar to those in [24][4], except that the Recursive least Squares (RLS) algorithm will be used rather than simple normalized projection, and tuning will be based on minimization of the input error rather than the output error. The advantage of this approach is that *only knowledge of plant order and delay is required for stability*, i.e., the requirements for prior partial Markov parameter information in Bayard [4] and for existence of Lyapunov equation solutions in Ortega [24] have been relaxed.

An added benefit of direct adaptive control is that numerical inversion of the estimate $\hat{H}$ of H is avoided. However, even though $\hat{H}$ is not inverted, its nonsingularity is still required to ensure adaptive stability. Hence, the polar decomposition introduced by Lozano will be needed to complete the stability proof.

Several simulation studies indicating the performance advantages of this direct adaptive approach relative to the earlier direct approaches in [24][4] can be found in Jakubowski et. al., [15][16].

Simply stated, the main contribution of this paper is to put stability requirements for convergence of direct adaptive periodic controllers on equal footing with requirements for indirect adaptive periodic control, as put forth Lozano [19]. The resulting stability conditions are simply that the plant order and delay are known a-priori. No prior knowledge of the plant high-frequency gain is used and persistent excitation is not required. More importantly, no assumption or knowledge is required as to whether the plant is minimum or nonminimum phase.

3 STABLE ADAPTIVE CONTROL

In this section, a stable direct adaptive controller is defined for the plant lifting (2.1).

3.1 Input Prediction Error

Given an estimate $\hat{\Theta}_{k-1}$ of Θ available at time k, one can construct the input prediction,

$$U_k^p = \hat{\Theta}_{k-1} r_k \tag{3.7}$$

and the associated input prediction error,

$$E_k \triangleq U_k^p - U_k = \Phi_{k-1} r_k \tag{3.8}$$

where,

$$\Phi_{k-1} \triangleq \hat{\Theta}_{k-1} - \Theta \tag{3.9}$$

3.2 Normalized Signals

For adaptation purposes, it is useful to define the following normalized quantities,

$$\tilde{r}_k = \frac{r_k}{1+\eta_k}; \quad \tilde{r}_k^d = \frac{r_k^d}{1+\eta_k}; \quad \tilde{E}_k = \frac{E_k}{1+\eta_k} \tag{3.10}$$

where the normalization factor is defined by,

$$\eta_k = ||Y_k|| + ||Y_{k-1}|| \tag{3.11}$$

Dividing through by η_k in (3.8) defines the normalized prediction error equation,

$$\tilde{E}_k \stackrel{\Delta}{=} \Phi_{k-1}\tilde{r}_k \tag{3.12}$$

3.3 Adaptation Algorithm

Equation (3.12') is a linear-in-the-parameter error expression for which many adaptation methods apply. The discussion here will focus on the Matrix Parameter Recursive Least Squares (MP-RLS) algorithm,

MP-RLS Adaptation Algorithm

$$\hat{\Theta}_k = \hat{\Theta}_{k-1} - \frac{\tilde{E}_k \tilde{r}_k^T F_{k-1}}{1+\tilde{r}_k^T F_{k-1}\tilde{r}_k} \tag{3.13}$$

$$F_k = F_{k-1} - \frac{F_{k-1}\tilde{r}_k \tilde{r}_k^T F_{k-1}}{1+\tilde{r}_k^T F_{k-1}\tilde{r}_k} \tag{3.14}$$

It is shown in [6] that the MP-RLS algorithm satisfies the following properties,

P1: $\Phi_k F_k^{-1} = \Phi_{k-1}F_{k-1}^{-1} = \dots = \Phi_0 F_0^{-1}$

P2: $v_k \le v_{k-1} \le \dots \le v_0$ where $v_k \stackrel{\Delta}{=} tr\{\Phi_k F_k^{-1}\Phi_k^T\}$

P3: $\bar{\sigma}(F_k) \le \bar{\sigma}(F_{k-1}) \le \dots \le \bar{\sigma}(F_0)$

P4: $\lim_{k\to\infty} \tilde{E}_k = 0$

P5: $tr\{\Phi_k \Phi_k^T\} \le v_0 \cdot \bar{\sigma}(F_0)$

P6: $\lim_{k\to\infty}(\hat{\Theta}_k - \hat{\Theta}_{k-1}) = 0$

P7: $\lim_{k\to\infty} F_k = F_\infty$

P8: $\lim_{k\to\infty} F_{k-1}\tilde{r}_k = 0$

P9: $\lim_{k\to\infty} \hat{\Theta}_k = \Theta_\infty = \Theta + \Phi_0 F_0^{-1} F_\infty$

3.4 Adaptive Control Law - Discussion

An adaptive control law is defined by replacing Θ in (2.5) by its estimate, i.e.,

$$U_k = \hat{\Theta}_{k-1} r_k^d \tag{3.15}$$

This control law is for discussion purposes only and will be modified subsequently.

Let the *output tracking error* be defined as,

$$\mathcal{E}_k = Y_k - Y_k^d \tag{3.16}$$

Using adaptive control law (3.15) and the MP-RLS estimator, the output tracking error is related to the input prediction error as follows,

$$\hat{E}_k = \frac{U_k^p - U_k}{1 + \eta_k} = \hat{\Theta}_{k-1}\hat{r}_k - \hat{\Theta}_{k-1}\hat{r}_k^d \tag{3.17}$$

$$= \frac{\hat{L}_{k-1}(Y_k - Y_k^d)}{1 + \eta_k} = \hat{L}_{k-1}\hat{\mathcal{E}}_k \tag{3.18}$$

where the normalized tracking error is defined as

$$\hat{\mathcal{E}}_k = \frac{\mathcal{E}_k}{1 + \eta_k} \tag{3.19}$$

Remark 1 For control purposes, it is desired for the output tracking error to converge to zero. Given that $\hat{E}_k$ goes to zero by property 1'4 of the estimator, it is clear from (3.18) that $\hat{\mathcal{E}}_k$ will also go to zero if $\underline{\sigma}(L_{k-1})$ is bounded away from zero. Unfortunately, while the true gain L satisfies this property, the estimate $\hat{L}_k$ produced from the recursive estimation scheme has no such guaranteed properties. The possible singularity of the estimate $\hat{L}_k$ destroys the above argument for convergence of the tracking error and is the *essence* of the difficulty associated with proving stability.

3.5 Adaptive Control Law - Modified

Lozano overcame the singularity problem for indirect adaptive control in [19] by introducing a modification of the matrix estimate based on a polar decomposition. A similar approach will be used here for direct adaptive control.

Consider the modified estimate,

$$\bar{\Theta}_k = \hat{\Theta}_k + R_k F_k \tag{3.20}$$

$$R_k = [0 \ | Q_k] \tag{3.21}$$

Here, matrix Q_k determined from a *polar decomposition*,

$$\hat{L}_k = Q_k S_k \tag{3.22}$$

where Q_k is a real orthogonal matrix, and $S_k = S_k^T \geq 0$ (cf., Barnett [3]). Note that the polar decomposition (3.22) is similar to the standard QR factorization, (i. e., where Q is orthogonal arid R is upper triangular), except that S_k is symmetric and non-negative definite. Conceptually, the polar decomposition can be written in terms of the singular value decomposition $L_k = U \Sigma V^T$ as follows,

$$\hat{L}_k = (UV^T)(V \Sigma V^T) \tag{3.23}$$

noting that $Q_k = UV^T$ is an orthogonal matrix and $S_k = V\Sigma V^T$ is symmetric non-negative definite by construction. The polar decomposition of a matrix gets its name from analogy to the polar decomposition of a complex number $z = e^{arg(z)}|z|$ since $S_k \geq 0$ plays the role of the nonnegative quantity $|z|$ and any unitary matrix Q can be written in the form e^{iW} with W Hermitian.

Using the modified estimate (3.20), a modified adaptive control law can be defined as,

Modified Adaptive Control

$$U_k = \bar{\Theta}_{k-1} r_k^d \tag{3.24}$$

$$\bar{\Theta}_{k-1} = [\bar{K}_{k-1} \mid \bar{L}_{k-1}] \tag{3.25}$$

where,

$$\bar{K}_{k-1} = \hat{K}_{k-1} + Q_{k-1} f_{k-1}^T m_{k-1} \tag{3.26}$$

$$\bar{L}_{k-1} = \hat{L}_{k-1} + Q_{k-1} f_{k-1}^T f_{k-1} \tag{3.27}$$

and m_{k-1} and f_{k-1} form the partitioned Cholesky factors of F_{k-1}, i.e.,

$$F_{k-1} = \mathcal{F}_{k-1} \mathcal{F}_{k-1}^T > 0 \tag{3.28}$$

$$\mathcal{F}_{k-1} = \begin{bmatrix} m_{k-1}^T \\ f_{k-1}^T \end{bmatrix} \tag{3.29}$$

3.6 Stability Results

The main result is given next

Theorem 1

Let the lifted plant (2.1) be controlled by the modified adaptive control (3.24) and MP-RLS estimation algorithm (3.13)(3.14), to follow the bounded trajectory $||Y_k^d|| \leq \kappa$. Then the signals U_k and Y_k, remain bounded and the tracking error goes to zero asymptotically, i.e.,

$$\lim_{k \to \infty} |Y_k - Y_k^d| = 0 \tag{3.30}$$

Proof: If the modified adaptive control (3.24) is applied to the plant (2.1) at each time k, the normalized input prediction error (3.10) becomes,

$$\tilde{E}_k = \frac{U_k^p - U_k}{1 + \eta_k} = \hat{\Theta}_{k-1} \tilde{r}_k - \bar{\Theta}_{k-1} \tilde{r}_k^d \tag{3.31}$$

$$= \hat{\Theta}_{k-1} \tilde{r}_k - \bar{\Theta}_{k-1} \tilde{r}_k^d + R_{k-1} F_{k-1} \tilde{r}_k \tag{3.32}$$

$$= \bar{\Theta}_{k-1} (\tilde{r}_k - \tilde{r}_k^d) - R_{k-1} F_{k-1} \tilde{r}_k \tag{3.33}$$

$$= \frac{\bar{L}_{k-1} (Y_k - Y_k^d)}{1 + \eta_k} - R_{k-1} F_{k-1} \tilde{r}_k \tag{3.34}$$

Taking the limit of both sides of (3.34) and applying (P4) and (P8) yields,

$$\lim_{k \to \infty} \frac{\bar{L}_{k-1} (Y_k - Y_k^d)}{1 + \eta_k} = 0 \tag{3.35}$$

Since by Lemma A2 of Appendix A, $\sigma(\bar{L}_{k-1}) > 0$ is bounded away from zero, it follows from (3.35) that.

$$\lim_{k\to\infty} \frac{\mathcal{E}_k}{1+\eta_k} = 0 \tag{3.36}$$

Note also that,

$$\frac{||\mathcal{E}_k||^2}{(1+\eta_k)^2} \geq \frac{1}{2} \frac{||\mathcal{E}_k||^2}{1+\eta_k^2} \tag{3.37}$$

where we have used the fact that $2\eta_k \leq 1 + \eta_k^2$. Combining results (3.36) and (3.37) it follows that,

$$\lim_{k\to\infty} \frac{||\mathcal{E}_k||^2}{1+\eta_k^2} = 0 \tag{3.38}$$

Now consider convergence of the unnormalized tracking error $\mathcal{E}_k$. Using the triangle inequality, one can verify the following linear boundedness condition,

$$\eta_k = ||Y_{k-1}|| + ||Y_k|| \leq ||Y_{k-1} - Y_{k-1}^d|| + ||Y_k - Y_k^d|| + ||Y_{k-1}^d|| + ||Y_k^d|| \tag{3.39}$$

$$= ||\mathcal{E}_{k-1}|| + ||\mathcal{E}_k|| + c_1 \leq c_1 + c_2 \max_{0\leq i\leq k} ||\mathcal{E}_i|| \tag{3.40}$$

where $c_1 = 2\kappa \geq ||Y_{k-1}^d|| + ||Y_k^d||$ and $c_2 = 2$. Given convergence of (3.38) and linear boundedness condition (3.40), the Key Technical Lemma (Goodwin and Sin [14]) ensures that,

$$\lim_{k\to\infty} \mathcal{E}_k = 0 \tag{3.41}$$

and that η_k remains bounded. Boundedness of η_k implies the boundedness of Y_k which together with P3, P5, and (3.24) imply the boundedness of U_k. ∎

Remark 2 In light of the discussion in Remark 1, the main idea behind the stability proof can be understood completely from (3.34). This relation uses the modified gain $\bar{L}$ and has the extra term $R_{k-1}F_{k-1}\hat{r}_k$ compared with equation (3.18), discussed earlier, This extra term is due to the modification (3.20) of the parameter estimate. Somewhat remarkably, this term vanishes by property P8 of the estimator. Since the modified estimate $\bar{L}_{k-1}$ is nonsingular by design (i.e., Lemma A2), the stability proof outlined in Remark 1 is recovered.

It appears that property P8 was first used for proving adaptive stability in the paper by Lozano and Goodwin [22], although the idea is implicit in an earlier paper by P. de Larminat [25]. In [22], P8 follows as a property of the normalized RLS algorithm with constant trace. Although the constant trace is dropped in the present MP-RLS algorithm, it can be shown (cf. [6]), that property P8 follows directly from convergence of the covariance matrix in property P7. ∎

4 CONCLUSIONS

It is shown that only knowledge of the plant order and delay is required to achieve stable direct adaptive control of nonminimum phase systems using periodic controllers. This relaxes requirements for stability found in earlier direct adaptive periodic control approaches involving the existence of solutions to Lyapunov equations [24], or partial plant Markov parameter knowledge [4].

As a result, stability requirements for convergence of direct adaptive periodic controllers are now on equal footing with requirements for indirect adaptive periodic control, as established in the work of Lozano [19].

Despite theoretical stability results, there are several open issues which remain to be resolved before the present approach can be made to work reliably in practice:

11. Reduction of adaptive transient

12. Modifications to meet actuator saturation constraints

13. Robustness to bounded process/measurement noise

14. Robustness to model Order/delay mismatch, unmodelled dynamics

Concerning 11 and 12, large transients are often experienced when simulating systems with adaptive periodic controllers. This is partly due to the *certainty equivalence* property of the adaptive control which is controlling the *wrong plant with conviction* most of the time. In addition, even the transient response in the nonadaptive case can be large due to the fast "inverse plant" nature of the control. Unfortunately, pole-placement strategies offer little relief since poles of the lifted system are associated with the slow time scale and hence must be kept near the origin to maintain reasonable performance. For the nonadaptive case, it has been shown in [5][6] that transients and control signals can be significantly reduced using extended horizon liftings. It is hoped that this same approach can lead to reduced transients in the adaptive case.

The algorithm in the present paper is not robust to bounded noise, and serves primarily to show equivalence of stability conditions between direct and indirect approaches under ideal conditions. Modifications similar to the deadzone in [19] are presently under consideration to address issue 13 in the direct adaptive case.

Issue 14 is perhaps the most difficult to address. The warnings contained in Goodwin and Feuer [13] regarding generalized sampling methods are most relevent for issue 14, since one must rely on high frequency plant dynamics for reliable control over low frequencies. A method proposed in Lozano [20] is applicable to overparametrization in the regulation problem, but presently has no extension to the tracking problem. Alternative approaches based on multiple model banks are emerging, and may play an important role in the future (c f., Morse [23]).

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A APPENDIX A: Supporting Results

The whole point of Appendix A is show that the modified gain $\underline{\sigma}(\hat{L}_k)$ is bounded away from zero. Lemma A1 is used to prove Lemma A2 which contains the final desired result.

Lemma A1 : Let the MP-RLS algorithm (3.13)(3.14) be applied to the normalized error equation (3.12). Then the estimate $\hat{L}_k$ and its polar factor S_k in (3.22) are explicitly bounded from above as follows,

$$\hat{L}_k \hat{L}_k^T \leq 2\alpha \cdot I \tag{A.1}$$

$$S_k \leq \sqrt{2\alpha} \cdot I \tag{A.2}$$

where,

$$\alpha = tr\{\Theta\Theta^T\} + v_0 \cdot \bar{\sigma}(F_0) \tag{A.3}$$

Proof: Consider the matrix inequality,

$$(X + Y)(X + Y)^T \leq 2(XX^T + YY^T) \tag{A.4}$$

Letting $X = \Theta$ and $Y = \Phi_k$ in (A.4) and using definition (3.9) gives,

$$\hat{\Theta}_k \hat{\Theta}_k^T \leq 2(\Theta\Theta^T + \Phi_k \Phi_k^T) \tag{A.5}$$

At this point, one can construct the following sequence of inequalities,

$$\hat{L}_k \hat{L}_k^T \leq \hat{\Theta}_k \hat{\Theta}_k^T \tag{A.6}$$

$$\leq 2(\Theta\Theta^T + \Phi_k \Phi_k^T) \tag{A.7}$$

$$\leq 2\, tr\{\Theta\Theta^T + \Phi_k \Phi_k^T\} \cdot I \tag{A.8}$$

$$\leq 2(\, tr\{\Theta\Theta^T\} + v_0 \cdot \bar{\sigma}(F_0)) \cdot I = 2\alpha \cdot I \tag{A.9}$$

Here, inequality (A.6) follows from the fact that $\hat{\Theta}_k \hat{\Theta}_k^T = \hat{K}_k \hat{K}_k^T + \hat{L}_k \hat{L}_k^T$; Inequality (A.7) follows from (A.5); Inequality (A.8) follows from the fact that $X \leq tr\{X\} \cdot I$ for any symmetric non-negative definite matrix X; Inequality (A.9) follows from property **P5** of the estimator; and definition of α in (A.3). This proves result (A.1).

Using the polar decomposition (3.22) in (A.9) gives the relation,

$$Q_k S_k^2 Q_k^T \leq 2\alpha \cdot I \tag{A.10}$$

Hence, for any vector y,

$$y^T S_k^2 y = y^T Q_k^T (Q_k S_k^2 Q_k^T) Q_k y \leq 2\alpha \cdot y^T Q_k^T Q_k y = 2\alpha\, ||y||^2 \tag{A.11}$$

where use has been made of (A.10) and the orthogonality property $Q_k^T Q_k = I$. Since y is arbitrary in (A.11), one can conclude that,

$$S_k^2 \leq 2\alpha \cdot I \tag{A.12}$$

which gives (A.2) upon taking the square root. •

Lemma A2: Let b_0 be a positive scalar such that,

$$L^T L > b_0 \cdot I > 0 \tag{A.13}$$

Let the MP-RLS algorithm (3.13)(3.14) be applied to normalized error equation (3.12). Then the gain modification defined by (3.20) ensures that ,

$$\bar{L}_k^T L_k \geq \frac{b_0}{\gamma^2} \cdot I > 0 \qquad (A.14)$$

where,

$$\gamma = 2 \max(v_0, \sqrt{2\alpha}) \qquad (A.15)$$

$$\alpha = tr\{\Theta\Theta^T\} + v_0 \cdot \bar{\sigma}(F_0) \qquad (A.16)$$

Proof: Define,

$$\beta_k = \Phi_k \mathcal{F}_k^{-T} \qquad (A.17)$$

Rearranging (A.17) and using (3.29) gives,

$$\Theta = \hat{\Theta}_k - \beta_k \mathcal{F}_k^T \qquad (A.18)$$

$$L = \hat{L}_k - \beta_k f_k \qquad (A.19)$$

Applying the matrix inequality,

$$(X - Y)^T(X - Y) \leq 2(X^T X + Y^T Y) \qquad (A.20)$$

with choices $X = \hat{L}_k$ and $Y = \beta_k f_k$ to (A.19) gives,

$$L^T L \leq 2 \cdot \left(\hat{L}_k^T \hat{L}_k + f_k^T \beta_k^T \beta_k f_k \right) \qquad (A.21)$$

At this point one can construct the following sequence of inequalities,

$$\begin{aligned}
L^T L &\leq 2\left(\hat{L}_k^T \hat{L}_k + f_k^T f_k \, tr\{\beta_k^T \beta_k\} \right) && (A.22)\\
&\leq 2\left(\hat{L}_k^T \hat{L}_k + f_k^T f_k \cdot tr\{\Phi_k F_k^{-1} \Phi_k^T\} \right) && (A.23)\\
&\leq 2\left(\hat{L}_k^T \hat{L}_k + f_k^T f_k v_0 \right) && (A.24)\\
&= 2\left(S_k^2 + f_k^T f_k v_0 \right) && (A.25)\\
&\leq 2\left(\sqrt{2\alpha} \cdot S_k + f_k^T f_k v_0 \right) && (A.26)\\
&\leq \gamma\left(S_k + f_k^T f_k \right) && (A.27)\\
&= \gamma Q_k^T \left(Q_k S_k + Q_k f_k^T f_k \right) = \gamma Q_k^T L_k && (A.28)
\end{aligned}$$

Here, inequality (A.22) follows from (A.21) by using the matrix inequality $X^T Y X \leq X^T X \, tr\{Y\}$ valid for any symmetric non-negative definite Y; Inequality (A.23) follows by using the definition of β_k in (A.17), Cholesky factors (3.28), and properties of the trace; Inequality (A.24) follows by property **P2** of the estimator; Equality (A.25) follows by substituting the polar decomposition (3.22); Inequality (A .26) follows by result (A.2) of Lemma A1; Inequality (A.27) follows by the definition of γ in (A. 15); and equation (A. 28) follows by the orthogonality of Q_k and the structure of the modified gain L_k in (3.27).

Using (A.13) and (A.28) gives upon squaring,

$$0 \le b_0^2 \cdot 1 \le (L^T L)^T (L^T L) \le \gamma^2 L_k^T Q_k Q_k^T L_k = \gamma^2 L_k^T L_k \tag{A.29}$$

Rearranging, gives the desired result (A.14). •

B APPENDIX B: Zero Annihilating Liftings

Consider the input/output model,

$$\mathcal{A}(z^{-1}) y_t = \mathcal{B}(z^{-1}) u_t \tag{B.1}$$

$$\mathcal{A}(z^{-1}) = 1 + \sum_{i=1}^{n} a_i z^{-i}; \quad \mathcal{B}(z^{-1}) = \sum_{i=1}^{n} b_i z^{-i} \tag{B.2}$$

where polynomials $\mathcal{A}$ and $\mathcal{B}$ are assumed to be relatively prime. It is assumed that $b_1 \neq 0$, so that the polynomial $\mathcal{B}$ can be factored uniquely into the form $\mathcal{B}(z^{-1}) = z^{-d} b_1 \bar{\mathcal{B}}(z^{-1})$ where $\bar{\mathcal{B}}(z^{-1})$ is monic and d = 1 is the plant delay. The choice $d = 1$ is for convenience only and is not a fundamental restriction. In the case that $d \neq 1$, all subsequent expressions can be appropriately modified without loss of generality.

Consider breaking the input and output signals into windows of length $N \ge 2n - 1$ where,

$$Y_f(k) = \begin{bmatrix} y_{kN+1} \\ y_{kN+2} \\ \vdots \\ y_{kN+N} \end{bmatrix}; \quad U_f(k) = \begin{bmatrix} u_{kN} \\ u_{kN+1} \\ \vdots \\ u_{kN+N-1} \end{bmatrix}$$

Albertos [1] has shown that the mapping between the vectorized quantities obeys the relationship,

$$Y(k) = A_a Y(k-1) + H_a U(k) + B_a U(k-1)$$

where matrices A_a, H_a, B_a can be written in terms of the elements of the polynomials $\mathcal{A}$ and $\mathcal{B}$.

It is useful to construct the "small" vector Y_k from certain components Y_f (k) as follows,

$$Y_k \stackrel{\Delta}{=} S_y Y_f(k) \in R^{o_y}$$

where $S_y \in R^{o_y \times N}$ is a *selection* matrix which sifts out o_y elements of $Y_f(k)$ for inclusion into Y_k.

The components of $Y_f(k)$ that are not put into Y_k define the co)//pl(T)/cnff7ry output vector $Y_k^c = S_y^c Y_f(k)$, with associated selection matrix S_y^c.

As a key step, it will be assumed that only certain components of $U_f(k)$ can be chosen nonzero. These components define the vector $U_k = S_u U_f(k)$ with associated selection matrix S_u. The fact that only selected components of U_f can be nonzero is mathematically stated as,

$$(I - S_u^T S_u) U_f(k) = 0 \tag{B.3}$$

The vectorized quantities Y_k, Y_k^c, U_k define a "lifted system". It can be shown ['1][5] that the lifted system has the following dynamics, *Lifted System Dynamics*

$$\begin{bmatrix} Y_k \\ Y_k^c \\ U_k \end{bmatrix} = \begin{bmatrix} S_y A_a S_y^T & S_y A_a (S_y^c)^T & S_y B_a S_u^T \\ S_y^c A_a S_y^T & S_y^c A_a (S_y^c)^T & S_y^c B_a S_u^T \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_{k-1} \\ Y_{k-1}^c \\ U_{k-1} \end{bmatrix} + \begin{bmatrix} S_y H_a S_u^T \\ S_y^c H_a S_u^T \\ I \end{bmatrix} U_k$$

Furthermore, for special choices of selection matrices S_u and S_y given in [4][5], the system dynamics (B) simplify to,

$$Y_k = S_y A_a S_y^T Y_{k-1} + S_y H_a S_u^T U_k \tag{B.4}$$

$$Y_k^c = S_y^c A_a S_y^T Y_{k-1} + S_y^c H_a S_u^T U_k \tag{B.5}$$

It is noted from (B.4)(B.5) that, the complementary signal Y_k^c no longer couples into the Y_k dyamics. Furthermore, the delayed input has vanished from the dynamics. With this structure, the transmission zeros from U_k to Y_k have been placed at the origin, and the class of liftings is said to be *zero annihilating.* General classes of such zero annihilating liftings were developed in Bayard [4][5], where Lozano's lifting [19] can be shown to be the special case $N = 2n$ and $\sigma_u = \sigma_y = n$. Furthermore, the matrix premultiplying the control U_k can be shown to be *full rank*, and hence is always invertible for control purposes (inversion is in a minimum-norm sense for nonsquare liftings $\sigma_u \neq \sigma_y$ [5]). The main importance of these liftings is that the lifted transfer function from U_k to Y_k is minimum-phase regardless of whether the original plant is minimum or non-minimum phase.

This paper focuses on adaptive control for the top equation (B.4), This is reasonable since (B.5) does not couple into (B.4), and the complementary signal Y_k^c is a bounded function of both Y_k and U_k. Indeed, any adaptive contr ol scheme which ensures boundedness of U_k and Y_k will ensure boundedness of the complementary signal Y_k^c.

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